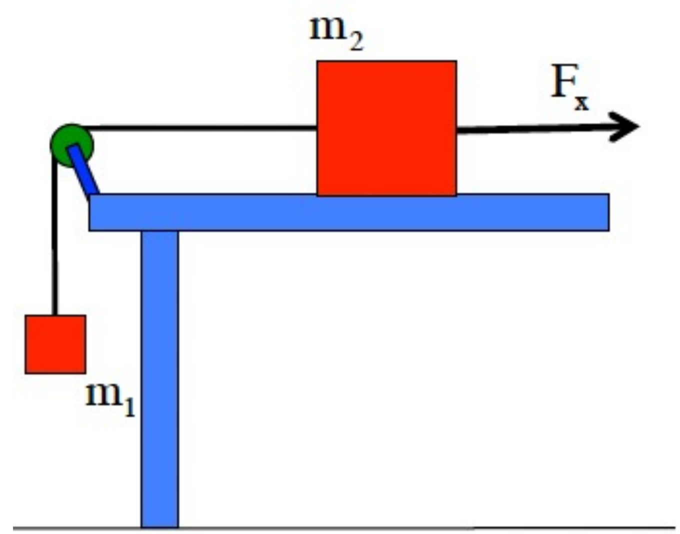
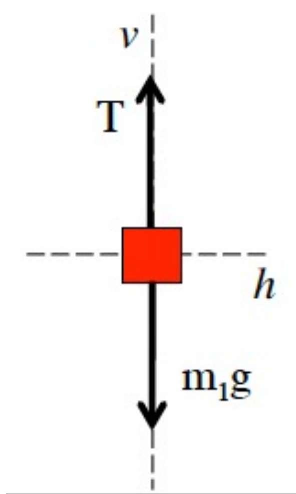
Problem 5.33

a.) For what value of "F" will the hanging mass accelerate upward?



Using N.S.L., the Formal approach, making the assumption that the hanging mass accelerates upward the the table mass accelerates to the right, we can write:

f.b.d on hanging mass:



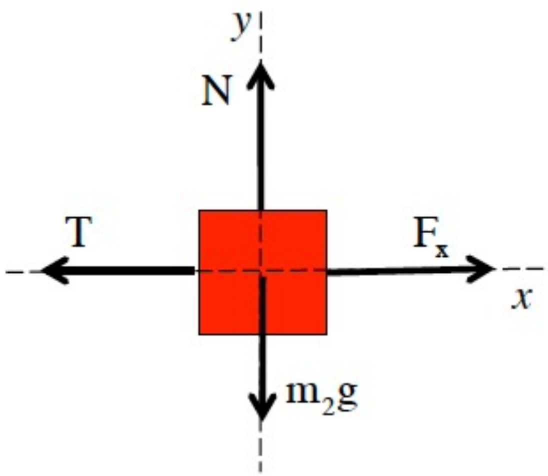
$\underline{\sum F_v:}$

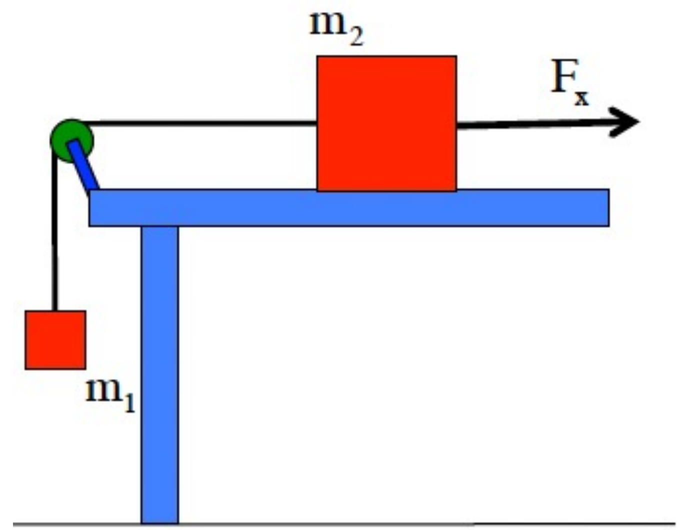
$$T - m_1 g = +m_1 a$$

$$\Rightarrow \quad T = m_1 g + m_1 a$$

For the mass on the table:

f.b.d. on m_2





and

$\underline{\sum F_x :}$

$$-T + F_x = m_2 a$$

$$\Rightarrow \quad -(m_1 g + m_1 a) + F_x = m_2 a$$

$$\Rightarrow \quad a = \frac{F_x - m_1 g}{m_1 + m_2}$$

This is an interpretation problem, and an educational one. We have assumed that the acceleration vector of the hanging mass is directed upward. We have unembedded the signs in our N.S.L. expressions accordingly, which means that when we solve for "a," we should get a positive number (after all, magnitudes are, by definition, positive). If we didn't, it would mean we had assumed the wrong direction for the acceleration, which is NOT the situation we want (that is, the situation in which the hanging mass accelerates DOWNWARD). Sooo, we need the "F" values that will allow our acceleration magnitude expression of

$$a_x = \frac{F_x - m_1 g}{m_1 + m_2}$$

to be positive. That will be the case as long as:

$$F_x > m_1 g \qquad \left(= (2.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 19.6\ \text{N}\right).$$

Using our two N.S.L. expressions to eliminate "a," we can get a relationship between "T" and "F." Doing so yields:

f.b.d. on hanging mass:

$$\underline{\sum F_v :}$$

$$T - m_1 g = +m_1 a$$

$$\Rightarrow \quad a = \frac{T}{m_1} - g$$

f.b.d. on tabletop mass:

$$\underline{\sum F_x :}$$

$$-T + F_x = m_2 a$$

$$\Rightarrow \quad -T + F_x = m_2\left(\frac{T}{m_1} - g\right)$$

$$\Rightarrow \quad T = \frac{F_x + m_2 g}{\left(1 + \frac{m_2}{m_1}\right)}$$

Apparently:

$$T = \frac{F_x + m_2 g}{\left(1 + \frac{m_2}{m_1}\right)}$$

According to our expression, "T" will be zero when:

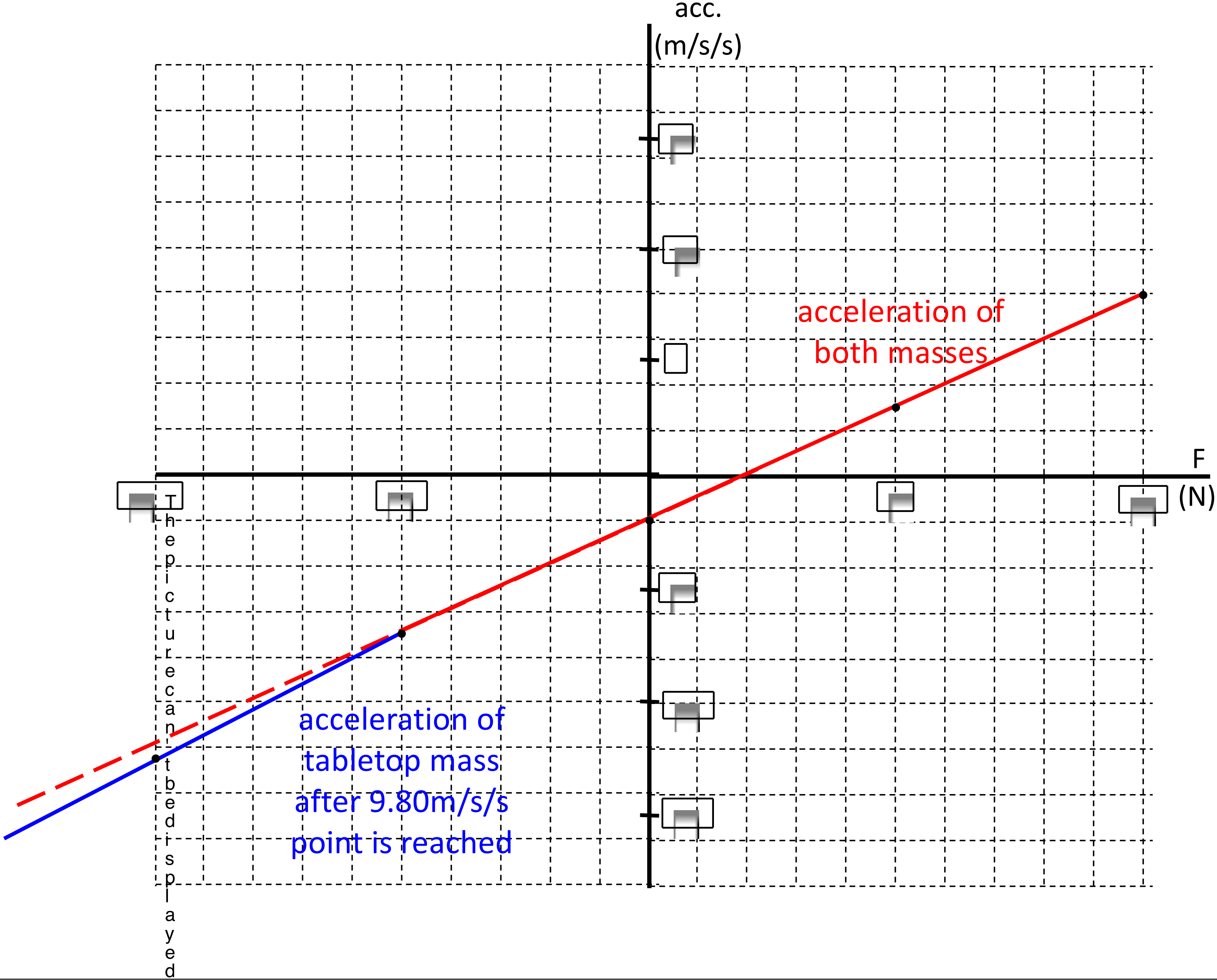
$$F_x < -m_2 g \qquad (= -(8.00\ \text{kg})(9.80\ \text{m/s}^2) = -78.4\ \text{N}).$$

That is what the math maintains, and it probably seems crazy until you think about it. The tension in the line will only happen when the acceleration of the hanging mass is greater than or equal to 9.8 m/s/s downward (that is, free fall). That will only happen if the force "F" is reversed in direction so that the tabletop mass accelerates at at least 9.8 m/s/s to the left (and if it accelerates at a rate greater than that, the hanging mass still free falls while the tabletop mass accelerates even faster). Crazy (as I said)? Yes, but also sensible if you think about it.

Using   for a range of F between +100 N and – 100 N, we get the following data:

F_x (N)	-100	-78.4	-50.0	0	50.0	100
a_x (m/s^2)	-12.5	-9.80	-6.96	-1.96	3.04	8.04

This data is graphed on the next page. Notice that the slope of the graph changes once you pass an acceleration of 9.80 m/s/s. This makes sense as up to 9.80 m/s/s the hanging mass and the table top mass have the same acceleration. Beyond 9.80 m/s/s the hanging mass free falls while the tabletop mass is motivated to higher and higher accelerations.


acc.
(m/s/s)
F
(N)
acceleration of
both masses
acceleration of
tabletop mass
after 9.80m/s/s
point is reached
The picture can't be displayed